The Frenkel Kontorova Model is a classical infinite chain of atoms linked by elastic springs with equilibrium spacing $a$ that is subject to an external periodic potential of period $b$ [1]. The model is characterized by the competition of the two different length scales, $a$ and $b$ and the lowest energy configurations of the chain have a very complex dependence on $a/b$ and the relative strength of the external potential, giving rise to commensurate or incommensurate configurations, and transitions between these, as the parameters of the model are varied [2,3]. The ground state configurations are also closely related to the unstable (hyperbolic) orbits of 2 dimensional hamiltonian maps such as the standard map, as follows from Aubry-Mather Theory [4].

It has been recently realized that a continuum hydrodynamic type evolution underlies the description of the static equilibrium state configurations. For the case of an elastic chain of particles embedded in an external potential, this evolution turns out to be governed by an iterated Burgers Equation [5,6] and the emerging shock discontinuities have a natural interpretation in terms of meta-stable states.

In this talk I will show that it is possible to treat the strong pinning limit exactly within the framework of kicked Burgers evolution. While recovering the results in [3], this approach also provides additional insights and results, such as the flow of shocks that allows for determining the structure of the meta-stable higher energy configurations, and the nature of the proliferation of the shock discontinuities at the phase boundaries where the configuration of the harmonic chain changes abruptly when external parameters are varied. Furthermore, these results also point the way towards perturbative treatments of the weak pinning limit and the effects of non-zero temperature. This work has been carried out in collaboration with Cem Yolcu [7].

References:


